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10MMD/MDE/MEA/MCM/MAR11

First Semester M.Tech. Degree Examination, December 2011
Applied Mathematics

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Define with suitable examples: i) Significant figure ii) Round off error iii) Truncation error iv) Absolute error. **(10 Marks)**

b. A parachutist of mass 68.1 kg jumps out of a stationary hot air ballon. Use $\frac{dv}{dt} = g - \left(\frac{c}{m}\right)v$ to compute velocity v prior to opening the chute. The drag coefficient is equal to 12.5 kg/s. Given that g = 9.8, v = 0 at t = 0. Apply finite divided difference scheme with a step size of 4 seconds for the calculation. **(10 Marks)**

2 a. Explain the bisection method to find the root of the equation f(x) = 0. Use it to find five approximations for $f(x) = x^3 - 5x + 1 = 0$ with four decimals in each computation. **(10 Marks)**

b. Use both the Newton-Raphson and modified Newton Raphson methods to find the real root near 2 of the equation $x^4 - 11x + 8 = 0$ accurate to five decimal places. **(10 Marks)**

3 a. Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^3 + x^2 - x + 2 = 0$. Use the initial approximations p = -0.9 and q = 0.9. **(10 Marks)**

b. Find all the roots of the polynomial $x^3 - 6x^2 + 11x - 6 = 0$ using the Graeffe's root squaring method. **(10 Marks)**

4 a. Given the following table of values, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.1 and x = 1.6, using suitable interpolation formula. **(10 Marks)**

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

b. Use Ramberg's method to compute $\int_0^1 \frac{dx}{1+x^2}$ correct to four decimal places. **(10 Marks)**

5 a. Explain the triangularisation method to solve the system of linear equations. Solve for $x_1 + x_2 + x_3 = 1$, $4x_1 + 3x_2 - x_3 = 6$ and $3x_1 + 5x_2 + 3x_3 = 4$ using the triangularisation method. **(10 Marks)**

b. Find the inverse of the matrix,

$$A = \begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}$$ using the partition method.

Hence, solve the system of equations AX = b where $b = [-10 \quad 8 \quad 7 \quad -5]^T$. **(10 Marks)**

6 a. Use Householder's method to reduce the matrix $\begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & -1 \\ 4 & -1 & 1 \end{bmatrix}$ to a tridiagonal matrix.

(10 Marks)

b. Using the Jacobi method, find all the eigen values and the corresponding eigen vectors of the matrix,

$$A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$$

(10 Marks)

7 a. Define a linear transformation T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ by $T(x) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}$. Find the images under T of $u = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $u + v = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$ **(06 Marks)**

b. Let T: $\mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation. Then T is one-to-one if and only if the equation T(x) = 0 has only the trivial solution. **(06 Marks)**

c. If T : $\mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation, prove that,

i) T maps R^n onto R^m iff the columns of A span R^m.

ii) T is 1 – 1 iff the columns of A are linearly independent. **(08 Marks)**

8 a. Let $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of y onto u. Then write y as the sum of two orthogonal vectors, one in span {u} and one orthogonal to u. **(08 Marks)**

b. Let W = span{x_1, x_2}, where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Construct an orthogonal basis {v_1, v_2} for W.

(04 Marks)

c. Find a least – squares solution of the system Ax = b for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$. **(08 Marks)**

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